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CONFIGURATION MIXING IN PREEQUILIBRIUM REACTIONS-A NEW LOOK AT THE HYBRID-EXCITON CONTROVERSY

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CONFIGURATION MIXING IN PREEQUILIBRIUM REACTIONS - A NEW LOOK AT THE HYBRID-EXCITON CONTROVERSY*

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Abstract

The physical concepts of the Hybrid and Exciton models are reexamined and shown to constitute fundamentally different approaches to prequilibrium reactions. The difference in cross section predictions obtained from the models is not attributable - as has often been argued - to inappropriate exciton distribution functions in higher order terms or multiple chance emission. It rather rests with the question of whether or not configuration mixing is assumed to take place during equilibration and what is assumed about hole interactions. A simplified but realistic example is given to illustrate these points, and a test against experimental data is proposed to decide which model is the more appropriate to use.

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1. Introduction

Nuclear reaction models to treat the equilibration phase of reactions leading to the formation of a compound nucleus have been around for many years.[1-8] Most of these models are semi-classical in nature and have been used with considerable success in describing experimental data pertaining to the equilibration process, mainly the forward peaked hard component observed in the continuous spectra of light ejectiles and the high energy "tails" seen in the excitation functions of activation cross sections. More elaborate quantum mechanical theories,[9-11] which are not easily applied to routinely calculate measurable preequilibrium cross sections, have tended, however, to support the foundations on which the semi-classical models are built. This has prompted continued interest in these models as tools both to predict cross sections for a number of practical purposes and to test the adequacy of the underlying physics.

Although quite a variety of model formulations and computational techniques have been employed, most approaches utilize - in one way or another - one or both of two basic concepts which stem from the "grandparents" of preequilibrium theory, the intranuclear cascade (INC) model of Goldberger.[1] and Griffin's statistical model of intermediate structure[2] (SMIS). The idea in the INC approach is to treat the equilibration processes as a series of quasi-free nucleon-nucleon scattering process in the nuclear environment and estimate the competition between such processes and the escape of excited particles into the continuum. Griffin's idea, on the other hand, was that equilibration is so complex a process that the occurence of configurations of single particle excitations capable of emission into the continuum may be estimated solely on a statistical basis and with little regard for the process that produces such configurations. The SMIS does not explicitly treat the competition between intranuclear collisions and escape into the continuum and can therefore not predict absolute emission cross sections, as the INC model does. It also involves an equal a priori probability assumption for all energetically possible configurations of single particle excitations at every stage of the equilibration process. It is, however, an extremely transparent model, and it's formulae can be evaluated on a hand calculator, as compared to the Monte Carlo technique required by the INC approach. Much effort has therefore been devoted to combining the advantages of both models into a single one, and two formulations have emerged that come very close to that goal. These are the Hybrid model[5] suggested by Blann and the Exciton model as formulated by Gadioli[6].

Both approaches compute the intranuclear transition rates on an a - priori basis (nucleon-nucleon quasi-free scattering) and treat the competition of emission into the continuum explicitely. Yet they yield closed formulae which are of a remarkably simple structure. This greatly facilitates calculations and has no doubt contributed to the popularity both models enjoy. The formulae also seem to reflect the reasoning on which the approaches are based in a suggestive manner, perhaps so much so that approximations and assumptions inherent in either model are sometimes overlooked. The formulae are not as transparent as they are simple, and they give different results. The longstanding[12,13] and continuing[14] debate as to which of the models is "correct" and why conflicting predictions for emission cross sections are obtained indicates that there is only incomplete understanding of the concepts on which the two models rest. It is the purpose of this paper to reexamine these concepts and illustrate the different physics and approximations that are employed. As it will turn out, neither model can be proven wrong on an a - priori basis and either ansatz is useful to explore the physics of the reaction process. It seems possible, however, to conduct a test against experimental data that will show with which concept nature happens better to agree.

For the sake of simplicity, both models are considered in their most basic form in this paper, i.e. without distinction between protons and neutrons and disregarding the extensive refinements that have been made over the years, such as inclusion of effects of the diffuse nuclear edge,[15] isospin conservation[16], cluster emission[17] and modifications[18] to the mean free path of nucleons in nuclear matter, to mention just a few. Futhermore, simplifications will be introduced about the energy dependence of intranuclear collision rates. It should be kept in mind that this is just for ease of analytic evaluation and illustration. No comparison with experimental data is given or intended, as this will require more rigorous numerical calculations.

In the literature, the term Exciton model is not uniquely applied to denote Gadioli's formulation[6]. In addition, it is sometimes used in reference to Griffin's SMIS as well as approaches[19,20] which are derived from it. These employ an average matrix element to calculate intranuclear collision rates, which is fitted to experimental data, rather than being computed from quasi-free scattering cross sections. While this may be a useful procedure to reproduce experimental data and serve the needs of applied physics, it is less suitable to test the underlying basic physics. This is because the nature of that average matrix element is not clearly defined and because preequilibrium emission cross sections are smooth, rather structureless functions of incident and ejectile energy as well as of the mass of the composite system. Consequently, even a large amount of experimental data will not readily overdetermine a free parameter such as an average matrix element to anywhere near a desirable degree, especially if it is assigned a mass and excitation dependence. For this reason, varieties of the Exciton model that contain such an adjustable matrix element will not be considered here, and the term Exciton model is exclusively applied to denote Gadioli's formulation[6] which is - as is Blann's Hybrid model - essentially parameter free.

Some of the points that will be discussed below are touched upon or are inherent in a preequilibrium model formulation given by Ernst et al.[13] in an attempt to reconcile the Hybrid and the Exciton approaches with one another. Their work will not be quoted in detail. Instead, the interested reader is referred to their paper.[13] It's main conclusion, however, namely that the Hybrid and the Exciton model can be reconciled once a proper book-keeping of exciton distributions is observed, is at variance with the results to be described below.

2. Physical and Computational Concepts

While the Exciton and Hybrid models constitute significantly different approaches to - and yield different results for - preequilibrium emission, they are still, to a large extent, built on the same basic assumptions concerning the physics of the reaction:

The fusion of target nucleus and projectile is assumed to result in the formation of an unequilibrated composite system of excitation E, in which only few (n_0-1) degrees of freedom participate in the excitation. These are envisioned to be single particle degrees of freedom and referred to as excitons, which may either be excited nucleons ("particles") or vacant single particle levels below the Fermi energy ("holes"). The equilibration of the system is then assumed to proceed via a series of two body collisions - hereafter called thermalizing collisions - between excited nucleons and nucleons below the Fermi energy. It is further assumed that the thermalizing collisions are of the Markoff type and that each one will create an additional particle-hole pair. Collisions reducing the number of excitons or leaving it unchanged are neglected. This approximation has been demonstrated[3,4] to be perfectly valid for the part of the equilibration phase that contributes significantly to precompound emission. It is, however, obviously a very poor one as thermal equilibrium is approached, so that neither the Exciton nor the Hybrid model can be expected to be suitable to treat evaporation. The requirement that a precompound model should include the evaporation limit is, on the other hand, neither a necessary condition for the model to be "correct", nor is it a sufficient one.

During the equilibration cascade, nucleons may occupy single particle levels at energies in excess of the particle's separation energy and Coulomb barrier. Whenever this occurs - hereafter called an emission chance - emission of the particle is possible and competes with further thermalizing collisions. The emission rates are calculated from the reciprocity theorem, and the rates at which thermalizing collisions takes place are derived from either quasi-free nucleon-nucleon scattering or the imaginary part of the optical potential, both approaches giving essentially the same results[21]. These rates, together with the assumptions about the equilibration process outlined above, serve as a common basis to both the Exciton and the Hybrid model.

The approaches also agree in that they group the emission chances which arise during the equilibration cascade into classes, each class corresponding to a term in the sum by which the preequilibrium emission cross section is eventually given in either formulation. The models differ fundamentally, however, in the way these classes are defined as well as in the physics that is envisioned to underly them and that will now be discussed.

2.1 The Hybrid Model Concept.

In the Hybrid model, preequilibrium emission within one class is given as a product of two factors. The first factor is the probability that one out of a total of n = p + h excitons sharing the total excitation E, is a particle residing at single particle energy e_p. The second factor is the (conditional) probability that it will then escape into the continuum rather than, and prior to, undergoing a thermalizing collision:

$$W_c\,(i,e_p) = \rho_{i,HM}\,(E,e_p,p,h) \times \frac{\lambda_c\,(e_p)}{\lambda_c(e_p) + \lambda_+(e_p)} \tag{1}$$

(See Table 1 for notation).

Obviously, the fate of a single exciton is considered in eq. (1). It's elevation to excitation e_p in the history of the equilibration cascade is contained in the first factor. By definition, this is normalized so that

$$\int_0^E \rho_{i,HM} (E,e_p,p,h)\, de_p + \int_0^E \rho_{i,HM} (E,e_h,p,h)\, de_h = p + h = n \qquad (2)$$

the total number of excitons in class i. Therefore, evaluating eq. (1) for all possible energies e_p, will cover emission chances of all p particles, although only one exciton is considered at a time. As the second factor in eq. (1) covers all emission chances the exciton under consideration offers prior to undergoing a further thermalizing collision, eq. (1) - evaluated for energies e_p - not only covers but also exhausts all emission chances which arise from all p particle excitons, until each of them participates in another thermalizing collision. Trivially, this applies to the h holes as well, since they can never lead to emission prior to undergoing a collision and thereby producing an excited particle. The number of excitons in class i assumed to either be emitted or undergo a thermalizing collision is

$$\int_0^E W_c\,(i,e_p)\, de_p + \int_0^E W_+\,(i,e_p)\, de_p + \int_0^E W_+\,(i,e_h)\, de_h = p + h = n \qquad (3)$$

$$\text{with} \quad W_+\,(i,e_{p/h}) = \rho_{i,HM}\,(E,e_{p/h},p,h) \times \frac{\lambda_+(e_{p/h})}{\lambda_c\,(e_{p/h}) + \lambda_+\,(e_{p/h})} \qquad (4)$$

(See Table 1 for notation)

Therefore, the structure of the Hybrid equation (1) implies that the model groups emission chances according to exciton generations. If, e.g., n_o excitons (p_o particles and h_o holes) are produced in the fusion of projectile and target, they form the first generation of excitons. All emission chances they offer are lumped into one class (class 0) and exhausted by evaluating eq. (1) with $p = p_o$, $h = h_o$ and for all energies e_p. Further possibilities for emission arise only from excitons which have participated in a thermalizing collision of first generation excitons, i.e. the second generation. It consists of all first generation excitons after they underwent a thermalizing collision (and thus changed their energy e_p, e_h) and their collision partners which were excited in the process. Again all emission chances which the second exciton generation offers are lumped into one class (class 1) and exhausted by evaluating eq. (1) with $p = p_1$, $h = h_1$, and for all energies $e_p \leq E$, and so on. The total precompound spectrum is then obtained by summing over all generations (classes), i.e.

$$\frac{d\sigma}{d\varepsilon} = \sigma_F \times \sum_i D_{i,HM}\, \rho_{i,HM}\, (E, e_p, p_i, h_i)\, \frac{\lambda_c(e_p)}{\lambda_c(e_p) + \lambda_+(e_p)} \tag{5}$$

$D_{i,HM}$ is a depletion factor which takes account of the fact that the probability of finding excitons in the second generation is reduced by particle emission from the first generation.

For all practical purposes, only very few terms of the sum have to be calculated, as - from generation to generation - the total excitation energy E of the system is shared among more and more excitons, and emission probabilities decrease rapidly. As each particle in a generation will lead to a 2 particle 1 hole subsystem in the next generation, and each hole to a 1 particle 2 hole subsystem, the daughter generation will comprise (except for depletion)

$$\begin{aligned} p_{i+1} &= 2p_i + h_i \quad \text{particles and} \\ h_{i+1} &= 2h_i + p_i \quad \text{holes.} \end{aligned} \tag{6a}$$

Consequently, a daughter generation comprises

$$n_{i+1} = p_{i+1} + h_{i+1} = 3n_i \tag{6b}$$

excitons, i.e., three times as many as the parent generation and not, as the original Hybrid formulation - missleadingly - suggests,

$$n_{i+1} = n_i+2 \quad . \tag{7}$$

The structure of eq. (1) implies more, however, than just the way in which emission chances are grouped. As the probability for a particle to escape into the continuum is expressed as a branching ratio of single particle rates, $\lambda_c\ (e_p)$ and $\lambda_+\ (e_p)$, pertaining to an exciton under consideration and of a given single particle excitation, e_p, excitons are assumed to have well defined energies between thermalizing collisions. This means that the n-exciton states through which the composite nucleus passes are envisioned to be combinations of single particle excitations e_1, e_2, ..., e_h which are independent of one onother except for the condition that

$$\sum_{k=1}^{n_i} e_k = E \quad . \tag{8}$$

Such combinations will be referred to hereafter as configurations, in the sense that the n-exciton wave function can be written as a product of single particle wave functions with eigenvalues e_k. The Hybrid model assumes that no "intrinsic" mixing of configurations is produced by the nuclear forces which can rather be entirely described by a potential well and two body collisions of independent excitons. So in an individual composite nucleus, the attainment of any two n-exciton configurations is mutually exclusive. The model does allow, however, for statistical configuration mixing in the trivial sense that the equilibration cascade may-alternatively-proceed through large numbers of different configurations. Thus, the probability $\rho_{i,HM}(e_p, p, h)$ of finding a particle at energy e_p - i.e. the probability that an n-exciton configuration with one particle at that excitation is attained - is an average over a large ensemble of "microscopically" different equilibration cascades. So while the Hybrid model does not group emission chances according to n-exciton states - but rather according to generations of independent excitons - it does imply that the n-exciton states are "pure" configurations and that no intrinsic configuration mixing occurs.

There is only a loose correspondence between the succession of generations in the Hybrid model and the time elapsed since formation of the composite system. In particular, excitons which are members of different generations may coexist at a given time. Owing to their independence, no exciton "knows", if, when, and how other excitons were emitted before - out of it's own generation or out of another - and if it is still in the original composite system rather than a daughter nucleus formed by a previous preequilibrium emission. Consequently, no distinction can be made in the Hybrid approach between single and multiple precompound emission. Instead, inclusive spectra are calculated with the approximation that all precompound ejectiles are emitted from the same composite system, and the number of particles emitted from generation i,

$$\int_0^E W_c(i,e_p)\, de_p \quad , \tag{9}$$

may even be larger than one. As long as multiple emission is unlikely (up to some tens of MeV of total excitation), this does not present a problem, and it certainly suits the many experiments in which inclusive spectra are measured. If multiple precompound emission is important, however, and if activation cross sections for specific nuclides are to be calculated, a Hybrid calculation is not adequate without additional consideration of the multiple chance emission problem[22)].

To evaluate the average probability $\sigma_{i,HM}$ $(E,e_p\ p,\ h)$ - often called the exciton densities or distribution functions, the Hybrid model approach employs Griffin's assumption that-on the average-each of the configurations which are possible in a system of n-excitons is attained with equal probability. Under this assumption, the well known Ericson[23] state densities

$$\omega\ (E,p,h) = \frac{g\ (g\ E)^{n-1}}{p!\ h!\ (n-1)!} \tag{10}$$

or a modification thereof [24] yield

$$\rho_{i,HM}\ (E,e_p,p_i,h_i) = \frac{g \cdot \omega\ (E-e_p,p_i-1,\ h_i)}{\omega\ (E,p_i,h_i)} \quad . \tag{11}$$

The Hybrid model uses

$$\begin{aligned} p_{i\ +\ 1} &= p_i + 1 \\ h_{i\ +\ 1} &= h_i + 1 \\ n_{i\ +\ 1} &= n_i + 2, \end{aligned} \tag{12}$$

instead of the relation (6) implied by grouping emission chances according to exciton generations. Moreover, eqs. (10) - (12) are not generally consistent with the nucleon - nucleon collision mechanism which the model assumes to mediate the transition between any two generations of excitons, and an incorrect depletion factor $D_{i,HM}$ is used in the original Hybrid formulation. Consequently, the use of eqs. (10) - (12) must be considered an approximation. While this is important from a conceptual point of view, the chosen approximations are very good for most of the practical model applications. A more detailed discussion of this point is given in Sections 3 and 4.

2.2. The Exciton Model Picture

In the Exciton model, preequilibrium emission within one class is given as the ratio

$$W_c(i,e_p) = \frac{\rho_{i,EM}\,(E,e_p,p,h)\cdot\lambda_c(e_p)}{\Lambda_{i,p}+\Lambda_{i,h}} \tag{13}$$

with

$$\Lambda_{i,p} = \int_0^E \rho_{i,EM}\,(E,e_p,p,h)\,[\lambda_c(e_p) + \lambda_+\,(e_p)]\,de_p$$

and

$$\Lambda_{i,p} = \int_0^E \rho_{i,EM}\,(E,e_h,p,h)\cdot\lambda_+\,(e_h)\,de_h \tag{14}$$

The number of excitons which - in each class - is assumed either to be emitted or to undergo a thermalizing collision is easily verified to be

$$\int_0^E W_c\,(i,e_p)\,de_p + \int_0^E W_+\,(i,e_p)\,de_p + \int_0^E W_+\,(i,e_h)\,de_h = 1 \tag{15}$$

with

$$W_+ (i, e_{p/h}) = \frac{\rho_{i,EM} (E, e_{p/h}, p, h) \cdot \lambda_+(e_{p/h})}{\Lambda_{i,p} + \Lambda_{i,h}} \tag{16}$$

Action of one and only one exciton is considered in each class, although any of the n-excitons is given the chance to play that role. As the thermalizing collision of one exciton will produce an additional particle hole pair, eq. (12) holds, and each Exciton model class covers the emission chances arising from all states of the composite system that have the same exciton number. The transition between any two n-exciton state generations is mediated by a thermalizing collision of one and only one exciton, increasing the exciton number by $\Delta n = 2$, and the total precompound spectrum is obtained by summing over all generations:

$$\frac{d\sigma}{d\epsilon} = \sigma_F \times \sum_i D_{i,EM} \quad \frac{\rho_{i,EM} (E, e_p, p_i, h_i) \cdot \lambda_c(e_p)}{\Lambda_{i,p} + \Lambda_{i,h}} \tag{17}$$

Again, only very few terms of the sum have to be calculated for practical purposes, as emission probabilities decrease rapidly with exciton number. Exciton numbers grow less rapidly ($n_{i+1} = n_i+2$) in the exciton model than they do in the Hybrid model ($n_{i+2} = 3n_i$). Consequently, eq. (17) converges more slowly than the corresponding Hybrid model expression (5).

As the depletion factor $D_{i,EM}$ is correctly computed in the framework of the Exciton model formulation [6], and as action of one and only one exciton is considered in each generation of eq. (10), the preequilibrium spectrum obtained is that of the first precompound particle out. The Exciton model produces exclusive spectra, as opposed to the inclusive spectra of the Hybrid approach. The calculation may, however, be extended to daughter nuclides to include multiple emission <u>without losing</u> the <u>distinction between single</u> and <u>multiple emission</u>.

The exciton distribution functions, $\rho_{i,EM}(E, e_{p/h}, p_i, h_i)$, are evaluated using the same equations (10 - 12) that the Hybrid model employs. In the framework of the Exciton model, too, they are inconsistent with the nucleon-nucleon scattering mechanism envisioned to mediate the transition from one n-exciton generation to the next. Again, they must be considered an approximation, as will be discussed in Section 3. They may be used, however, to demonstrate an additional fundamental implication of eq. (13): Substitution of eq. (11) into (13) yields

$$W_c(i,e_p) = \frac{g \cdot \omega(E - e_p, p_i - 1, h) \cdot \lambda_c(e_p)}{\int_0^E g \cdot \omega(E - e, p_i - 1, h_i)\,[\lambda_c(e_p) + \lambda_+(e_p)]\,de_p + H}, \qquad (18)$$

where H is an analogous term for holes, and $\omega(E-e_p, p_i-1, h_i)/g$ is the number of (distinguisbable) configurations[23,24] of p_i-1+h excitons and energy $E-e_p$. So the numerator in eq. (18) comprises the rates of emitting a particle of energy e_p for those $n_i = p_i + h_i$ exciton configurations in which one particle is excited to energy e_p. They compete with the rates comprised in the denuminator, namely those of either emission or thermalizing collision of <u>all</u> excitons (at any energy $e_{p/h}$) for <u>all</u> n_i - exciton configurations possible at total excitation E, because

$$\int_0^E \omega(E - E_p, pi - 1, h_i)\,de_p + H = n \cdot \omega(e, p_i, h_i) \quad . \qquad (19)$$

In particular, the emission of a particle at excitation e_p from a configuration containing such a particle competes against the decay of configurations in which <u>all</u> excitons are excited to energies <u>other</u> than e_p. This is impossible if the configurations are assumed to be "pure" as in the Hybrid model. Therefore, the Exciton model implies thorough intrinsic configuration mixing, caused by a part of the nuclear Hamiltonian represented neither by the nuclear well nor by quasi-free nucleon-nucleon collisions, and the Exciton model exciton distribution functions $\sigma_{i,EM}$ must be interpreted as the average statistical weight which the "pure" configurations carry in the "real" n_i-exciton wave functions. This is to be contrasted to the ensemble average character of the corresponding (and numerically identical) distribution function $\sigma_{i,HM}$ in the Hybrid model concept.

2.3 Important and Less Important Differences.

Of the differences between (and the approximation used in) the Hybrid and the Exciton models as outlined above, some will be shown to be relatively unimportant in practical applications by the realistic example given in Section 4. These differences will include the question of multiple versus single preequilibrium emission and the approximations used for the exciton distribution functions in higher order (i>o) terms of either model. The single difference between the approaches, which is of the foremost significance <u>practically</u> and <u>conceptually</u>, is that of <u>zero</u> versus <u>maximum</u> intrinsic configuration mixing. It is also, perhaps, the most elusive one and worth demonstrating in an (unrealistic but) illustrative example, as depicted in Fig. 1. Consider a 2 particle 0 hole system in which only the two configuration denoted A (open circle excitons) and B (full dots) are energetically possible. Assume that they are attained with equal probability (in the Hybrid picture) or carry equal average statistical weight in the 2 exciton wave functions (in the language of the Exciton model). Then the probability of finding a particle at the highest possible single particle level (i.e. just above the emission threshold) is

$$\sigma_{i,EM}\,(E,e_{m},2,0) = \sigma_{i,HM}\,(E,e_{m},2,0) = 0.5 \quad ,$$

and it is the same for any of the other levels. Assume further that the escape and collision rates λ_c, λ_+ pertaining to the various single particle levels are as indicated in the figure in arbitrary units. Then the Hybrid model predicts a probability of

$$W_c\,(i,e_m) = 0.5 \times \frac{10}{10 + 10} = 0.25 \tag{20}$$

(see eq. (1))

for particle emission, while the Exciton model will yield

$$W_c\,(i,e_m) = \frac{0.5 \cdot 10}{0.5 \cdot 10 + 0.5 \cdot 1000 + 0.5 \cdot 1000 + 0.5 \cdot (10 + 10)} \simeq .005 \tag{21}$$

(see eq. (13)).

The striking difference in the model predictions is entirely due to opposing assumptions about configuration mixing which are employed. In the Hybrid model, no intrinsic configuration mixing is assumed. So in 50% of all equilibration cascades in an ensemble, configuration A is attained and suffers no interference from the existence of configuration B as a possible alternative. In configuration A, the higher energy particle has a 50% chance of escape (irrespective of what the lower energy particle will do), and the total Hybrid prediction for emission is just the product of these two independent probabilities. In the Exciton model, on the other hand, strong intrinsic configuration mixing is assumed, so that the "real" 2-exciton wave functions are linear combinations of the configurations A and B, each of which contributes with the same average strength. Consequently, each "real" 2 exciton state has a chance to decay through it's configuration B component, and it will do so with overwhelming probability, as the collision rates λ_+ associated with configration B are so large. Configuration A - although it offers a 30% escape chance for emission of the higher energy particle when considered seperately in the Exciton model framework - does not contribute appreciably to the average 2 - exciton state decay. It's contribtuion to the total average decay rate is only

$$0.5 \cdot (10 + 10) + 0.5 \cdot 10 = 15 \quad ,$$

(22)

as opposed to

$$0.5 \cdot 1000 + 0.5 \cdot 1000 = 1000$$

(23)

for configuration B.

Assume now, that the collision rates indicated in Fig. 1 are changed to be $\lambda_+ = 1$ for all single particle levels. Under this assumption, the Hybrid model prediction changes to

$$W_c\ (i,e_m) = 0.5 \times \frac{10}{10 + 1} \approx 0.45 \quad , \tag{24}$$

while the Exciton model gives

$$W_c\ (i,e_m) = \frac{0.5 \times 10}{0.5 \times 14} \approx 0.71 \tag{25}$$

The probability of emitting a particle of energy e_m in the Exciton model frame work is thus seen to be possibly greater than the probability of finding it at that excitation in the first place. Furthermore, the Exciton model result was changed by two orders of magnitude versus only a factor of two in the Hybrid result. Now add a hole to configuration A and assign it a transition rate of $\lambda_+ = 2000$. That leaves the Hybrid result unchanged, whereas the Exciton model prediction drops to $W_c(i,e_m) = 0.01$. The decay of the 2-exciton states now proceeds predominantly through this configuration A component, but via interaction of a hole, which is, of course, incapable of nucleon emission.

Obviously, none of the examples just studied resembles a real nucleus. They serve to illustrate, however, the effects which intrinsic configuration mixing can have on a preequilibrium calculation. Whether or not such configuration mixing is assumed is of great conceptual significance, as it makes for the difference between the purely quasi-free scattering picture of the Hybrid model approach and an additional part of the nuclear interaction, which is assumed to produce intrinsic configuration mixing. It is also, however, of great practical importance. As is familiar to practitioners of both models and has been pointedly stated by Chiang and Hüfner[8], the first term in eqs. (5) and (17) strongly dominates the high energy part of the preequilibrium spectrum. Most of the differences between the models which have been outlined in subsections 2.1 and 2.2 do not (or only marginally so) affect this first term. Intrinsic configuration mixing, on the other hand, does affect the first term, and it is particularly the hole interaction which plays a crucial role. This will be demonstrated in the example given in Section 4. The example will also show that inconsistencies from which both models suffer with respect to higher terms ($i>o$) have only a small effect on the predicted spectra compared to the influence exerted by intrinsic configuration mixing.

3. Exciton Distribution Functions

Both in the Hybrid and in the Exciton model, exciton distribution functions are assumed to be given by eqs. (10) - (12), although they have different meanings in either model and are inconsistent with the way the Hybrid model groups emission chances. Moreover, eqs. (10) - (12) require that all possible n-exciton configurations be populated with the same probability. Blann[22,25] has shown that all possible (n + 2) - exciton configurations are accessible through quasi-free nucleon-nucleon scattering from all (but not each of the) n-exciton configurations. They are not populated with equal probability, however, with one notable exception. If a single particle exciton scatters off any nucleon below the Fermi surface, the exciton distribution function for the resulting 2 particle 1 hole system will be, within about 20% error margin,

$$\rho\ (e_p, e_p', 2,1) = g \cdot \frac{4\ (e_p - e_p')}{e_p^2} \tag{26}$$

i.e. in agreement with eqs. (10) - (12), evaluated for total excitation e_p, $p=2$ and $h=1$. This was demonstrated by Blann[22,25] in a quasi-free scattering calculation. His result can be used to calculate (within the accuracy of eq. (26)) the exciton distribution functions in both the Hybrid and the Exciton model framework, which should be used instead of eqs. (10) - (12). Suppose in some generation i (e.g. in $i = 0$) the distribution functions are given by eqs. (10) - (12) and that they comprise $n_0 = p_0 + h_0$ excitons. Then the distribution functions for the next generation ($i = 1$), i.e. the probabilities of finding a particle at energy e_p' are

$$D_{1,HM}\,\rho_{1,HM}\,(E,e_p',\ 2p+h,\ 2h+p) =$$

$$= \int_{e_p'}^{E} \rho o,HM\,(E,e_p p,h) \cdot \frac{\lambda_+(e_p)}{\lambda_+(e_p) + \lambda_c(e_p)} \cdot \rho\,(e_p,e_p',2,1)\,de_p + H \quad (27)$$

in the Hybrid model and

$$\rho_{1,EM}\,(E,e_p',\ p+1,\ h+1) =$$

$$+ \int_{e_p'}^{E} \frac{\rho_{o,EM}\,(E,e_p',p,h) \cdot \lambda_+(e_p)}{\Lambda_{o,p} + \Lambda_{o,h}} \cdot \rho(e_p,e_p',2,1)\,de_p +$$

$$+ \int_{o}^{E-e_p'} \frac{\rho_{o,EM}\,(E,e_p',p,h) \cdot \lambda_+(e_p)}{\Lambda_{o,p} + \Lambda_{o,h}} \cdot \rho_{o,EM}\,(E-e_p,e_p',p-1,h)\,de_p + H \quad (28)$$

in the Exciton model. In either equation, only the part of the particle distribution arising from particle scattering is explicitely written. The part H which arises from hole scattering is analogous and must be added. Completely analogous expressions are valid for the hole distribution in class 1. Note that the integration limits ensure energy conservation for each possible nucleon-nucleon scattering process and that the distributions are normalized to

$$\int_{0}^{E} \rho_{1,HM}\,(E,e_p',\ 2p+h,\ 2h+p')\,de_p = 2p + h \quad (29)$$

and

$$\int_{o}^{E} \rho_{1,EM}\,(E,e_p',\ p+1,\ h+1) = p + 1 \quad (30)$$

in accordance with the different kinds of generations into which emission chances are grouped in the models. The Exciton model depletion factor is

$$D_{1,EM} = 1 - \int_0^E W_c(i,e_p)\, de_p \quad , \tag{31}$$

as in the original formulation.[6] In eq. (28), the first term is the contribution arising from the particles that participated in the collision mediating the transitions from n-exciton states to those with n + 2 excitons, whereas the second term covers the contribution of particles that remained spectators to that collision. This second term contains the probability $\sigma_{o,EM}(E-e_p, e_p', p-1, h)$ that after one particle of energy e_p is singled out to undergo a thermalizing collision, the rest of the system contains another particle at energy e_p'. If all configurations are equally likely in class 0, as was assured to be the case here, it is readily evaluated according to eqs. (10) - (12). In general, however, it is more tedious to calculate and becomes increasingly complex as one goes on to further generations. It also prevents eq. (28) from becoming truly recursive, as is the corresponding Hybrid equation (27).

Expressions analogous to (28) can be written for exciton distributions in daughter nuclides produced by precompound emission in the Exciton model framework. They also become increasingly lengthy the further one follows the chain of thermalizing collisions and emission processes. In principle, however, eqs. (27) and (28) provide the recipe to substitute the currently used expressions (10) - (11) with more exact approximations which are consistent with two body collisions and the way in which emission chances are grouped in the Hybrid and the Exciton models.

4. A Realistic Example of Model Intercomparison and Sensitivity to Assumed Transition Rates.

In order to assess the practical significance of the differences between the Exciton and the Hybrid models and of the approximations employed, a new realistic numerical example will now be discussed. It is a simplified case and not intended for comparison with experimental data, but it is realistic in that reasonable or reasonably demonstrative transition rates and functional dependences are used. It is meant to show what sort of effect results on a typical preequilibrium calculation as consequences of the points which were discussed from a conceptual point of view in Sections 2 and 3.

The example chosen is that of a 30 MeV nucleon incident on a medium mass nucleus with a nucleon separation energy of 7 MeV. The single particle escape and transition rates were taken to be those depicted in fig. 2, the transition rates approximating Gadioli's[24] calculation and the escape rates approximating a crude reciprocity expression. Alternative transition rates for holes are also considered (dashed and dotted lines in fig. 2). The full line in Fig. 2 will be referred to as strong hole interaction ($\lambda_+ (e_h) \alpha\, e_h^2$), whereas the dashed ($\lambda_+ (e_h) \alpha\, e_h$) and dotted ($\lambda_+ (e_h) = o$) lines will be called medium and zero hole interaction, respectively.

The assumption of equal a-priori probability is justified[22,25] for all 2 particle 1 hole configurations possible resulting from target-projectile fusion of an incident nucleon. Nucleon emission from the first 2 particle 1 hole generation (i=0, n=n_o) is readily calculated. Figure 3 shows the result obtained in the Exciton model for zero (dotted line), medium (dashed line) and strong (full curve) hole interaction. The results vary by a factor of 4, depending upon the hole interaction which is assumed, and Fig. 4 illustrates the reason for these pronounced differencs: In the case of strong hole interaction (full line) the initial 3-exciton states decay predominantly via hole-hole interactions, reducing particle emission accordingly. If, on the other hand, zero hole interaction is assumed (dotted line in Fig. 4), the 3-exciton states decay exclusively by particle emission or particle-particle intranuclear collision, and the probability of either is correspondingly high (Figs. 3, 4, dotted line). Medium hole interaction - produces an intermediate result both for particle emission and for a particle thermalizing collision to occur (dashed lines in Figs. 3,4). In Fig. 5, the preequilibrium emission probability from class 0 - i.e. the first term in eqs. (5) and (17) - in the Hybrid and Exciton models are compared with one another. The Exciton model result is seen to vary by about a factor of 4, as the assumption about hole interactions is changed from zero to strong (dotted and full curve), whereas the Hybrid model result is the same under both assumptions. In the high energy part of the spectrum, which is most important for comparison with experimental data due to precompound decay, the Exciton model is seen to predict emission probabilities (full curve) which are down by a factor of about two from the Hybrid model result if strong hole interaction is assumed. On the other hand, if no hole interaction is assumed in the Exciton model, (dotted curve) larger preequilibrium emission probabilities result than are obtained from a Hybrid calculation at high ejectile energies. These differences are almost exclusively due to the influence of intrinsic configuration mixing envisioned by the Exciton model as opposed to no mixing in the Hybrid model concept.

Under the assumptions used in this example for the transition rates, the integrals (27) and (28) can be solved analytically to yield the second generation exciton distribution functions which are shown in Fig. 6. Inspection of the figure shows that the rather accurate results obtained from eqs. (27) and (28) don't depend very much on what is assumed about hole interaction in either model. The figure also shows that the Hybrid distribution functions are much softer than those obtained in the Exciton model and that both are much softer than the first generation distribution function which is the same in either model. In terms of high energy ejectile emission this means that the second term in the Exciton model, eq. (17), contributes only a fraction of the first term, and that in the Hybrid model that fraction is still smaller. If eqs. (10) - (12) were used to calculate the second generation distribution functions, they would over-or under-predict the more exact results of eqs. (27) and (28) by factors $F_>$ and $F_<$, respectively which are shown in Fig. 7. at the maximum particle energies, e.g. eqs. (10) - (12) will give an exciton density four times as large as calculated with eq. (13) in the Hybrid model. While these differences are serious, they are seen (Fig. 7) to be of no great importance to high energy preequilibrium emission. Here, the probabilities of preequilibrium emission from the second generation are given as a fraction of emission from the first generation, $W_c(e_p)$. For the Exciton model, all three assumptions on hole interaction yield essentially the same result.

For particle energies above 25 MeV, the second term contributes about 40% or less of what the first term yielded, and second chance emission is even less important. Terms other than i=o are then seen to affect mostly the lower ejectile energies, for which eqs. (10) - (12) are a good approximation according to Fig. 7. Consequently, using eq. (28) instead of (10) - (12) will leave the Exciton model prediction for the preequilibrium spectrum essentially unchanged. In the Hybrid model the second generation contribution is even less significant than in the Exciton approach, as is seen in Fig. 8. Only in roughly the lower half of the emission spectrum will it give any appreciable contribution. Therefore, using eq. (27) instead of using the simpler Emission state densities (eqs. (10) - (12)), will change the result of a Hybrid calculation only marginally.

As emission chances are grouped according to different kinds of generations in either approach, no rigorously meaningful comparison can be made between contributions of individual generations of different models. Nevertheless, a rough comparison was made in Fig. 5 on the pretext that the first term plays a dominating role and can be used, perhaps, as a zero order approximation to a full calculation. More nearly equivalent to the Hybrid first generation is the sum of the first and second generation and of second chance emission in the Exciton model. This comparison is made in Fig. 9. It covers chances up to the point that two thermalizing collisions have occured in either model, and that a maximum of two particles could have been emitted. Inspection of the figure shows that the shapes of the spectra which are predicted, are more similar to one another than they are in Fig. 5. The difference in absolute magnitude visible at the high energy and in Fig. 5 are seen to persist in practically the entire spectrum when the "fairer" comparison shown in Fig. 9 is made.

5. Conclusions

The Exciton model and the Hybrid model have been shown to differ fundamentally in several ways. The Hybrid approach groups emission chances according to generations of independent excitons and yields inclusive spectra. The Exciton model groups emission chances according to generations of n-exciton configurations and yields exclusive spectra. It is a systems rather than an independent particle approach. Both models use the same closed form expressions as exciton distribution functions. These are inconsistent with two-body thermalizing collisions in the framework of either model. More accurate and consistent exciton distribution functions were given (Section 3) but shown to have only marginal impact on the results of a simplified but realistic calculation. This finding is expected to be generally valid and is due to the overwhelming importance of first generation emission, which is not affected by the approximations made for higher terms. Exceptions may possibly be reactions where first generation emission is suppressed by the nature of the entrance channel, e.g. proton preequilibrium emission induced by capture of negative pions.

The difference between the models, however, which is by far the most important - conceptually and numerically - is that no intrinsic configuration mixing is assumed in the Hybrid model, whereas the Exciton model implies strong mixing. This mixing, which is restricted to occur only among configurations of the same exciton number, affects the (dominating) first generation emission. As a consequence, an Exciton model calculation is very sensitive to what assumption is made about the interaction of holes.

There is no obvious a priori basis on which to estimate the amount of configuration mixing likely to occur during equilibration. It is incompatable with a pure two-body collision concept, as is would require collisions which leave the exciton number unchanged and these can easily be estimated to be very unlikely. Rather, it must be produced by a part of the nuclear Hamiltonian which is not described by the potential well and two-body collision. In addition, the question of hole interaction has not been studied sufficiently well to base a decision between the model concepts on a comparison of absolute cross sections to experimental data. As - unlike the Hybrid model predictions - the Exciton model results will strongly depend on hole interaction, agreement or disagreement with experimental data may just reflect the choice of a favorable or unfavorable hole interaction. The models differ, however, in the trend the preequilibrium spectra follow as a function of the total excitation. This trend is only partly influenced by the hole interaction and can most likely be used to decide whether or not there is configuration mixing in preequilibrium processes. If ratios of higher ejectile energy emission cross sections obtained with a number of different projectile energies are considered, the uncertainty resting with hole interaction assumptions is considerably reduced. In addition, the question of hole interaction might be studied in the same way Blann has used to justify eq. (26).

The difference in preequilibrium emission cross section predictions obtained from the Hybrid and Exciton models rests almost exclusively with the question of instrinsic configuration mixing. Past comparisons, which have led to adverse conclusions about the mean free path of nucleons in nuclei, were affected by differences in the single particle state densities used and also suffered from inconsistencies beyond the conceptual difference of the models.

Once the latter is recognized (and, perhaps, decided) and the former eliminated, a unique set of mean free paths may be shown to result from preequilibrium analysis.

Table 1. Definition of Symbols

p,h	Number of particles or holes
e_{ph}	Single particle (or hole) excitation, measured from the Fermi energy
E	Total excitation energy in the composite system
$\lambda_c(e)$	Escape rate of a particle of excitation e into the continuum
$\lambda_+(e)$	Rate, at which a particle (or hole) of excitation e undergoes a thermalizing collision
B	Separation energy
ε	Ejectile channel energy
σ_F	Entrance channel fusion cross section
$\sigma_{i,\binom{HM}{EM}}(e)$	Probability of finding an exciton at excitation e in the i'th generation in the Hybrid or Exciton model
i	Generation index
g	Single particle level density
$W_c(i,e_p)$	Probability of emitting a particle of energy e_p into the continuum
$D_{i,\binom{HM}{EM}}$	Depletion factor
$\frac{d\sigma}{d\varepsilon}$	Preequilibrium emission cross section
$\omega(E,p,h)$	Particle hole level density
$\lambda_{i,h}^{p}$	n exciton state average decay rate with respect to particle/hole transitions

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Fig. 1. An illustrative (but unrealistic) example for zero versus strong configuration mixing. The only two possible, equally likely configurations A and B are shown. The consequences of different assumptions for λ_c and λ_+ for Exciton and Hybrid models are disscussed in the text.

Fig. 2. Single particle escape and transition rates used in the demonstrative, realistic example in this work. The curves or lines represent rates adopted and several alternatives used for hole transitions, while the points indicate results of the detailed calculations.[24] The crosses correspond to results assuming a truncated harmonic oscillator potential end the open circles correspond to results assuming a Fermi gas with 20 MeV Fermi energy.

Fig. 3. First chance (class '0') preequilibrium emission probabilities obtained from the Exciton model for various assumptions about hole interactions. The dotted, dashed and full lines correspond to curves for hole-hole interactions in Fig. 3.

Fig. 4. Probabilities for decay of 2p1h states through a thermalizing collision of an exciton of energy $e_{p/h}$, as obtained in the Exciton model. The different curves pertain to the different assumptions about hole interactions which are shown in Fig. 3.

Fig. 5. First term (class 0) preequilibrium emission probabilities. Exciton model results for no hole interactions (dotted line) and strong hole interactions (full curve) are compared to a Hybrid model result (dash-dotted curve), which is independent of the hole interaction assumption.

Fig. 6. Exciton distribution functions resulting from eqs. (27) and (28) for the second generation ($i = 1$) of emission chances under zero, medium and strong hole interaction assumptions (dotted, dashed and full lines respectively). For comparison, the first generation ($i = o$), $n = n_0$) distribution function is also indicated (upper full curve).

Fig. 7. Factors by which eqs. (10) - (12) will overpredict ($F_>$) or under-predict ($F_<$) the more exact second generation exciton densities obtained with eqs. (27) and (28) in both the Hybrid and the Exciton model frameworks.

Fig. 8. Second generation (and second chance, in the Exciton model) preequilibrium emission as a fraction of first generation emission.

Fig. 9. Emission probabilities obtained from roughly comparable parts of Hybrid - and Exicton model calculations. The first generation Hybrid term is compared to the sum of first and second generation (and second chance) emission in the exciton model.

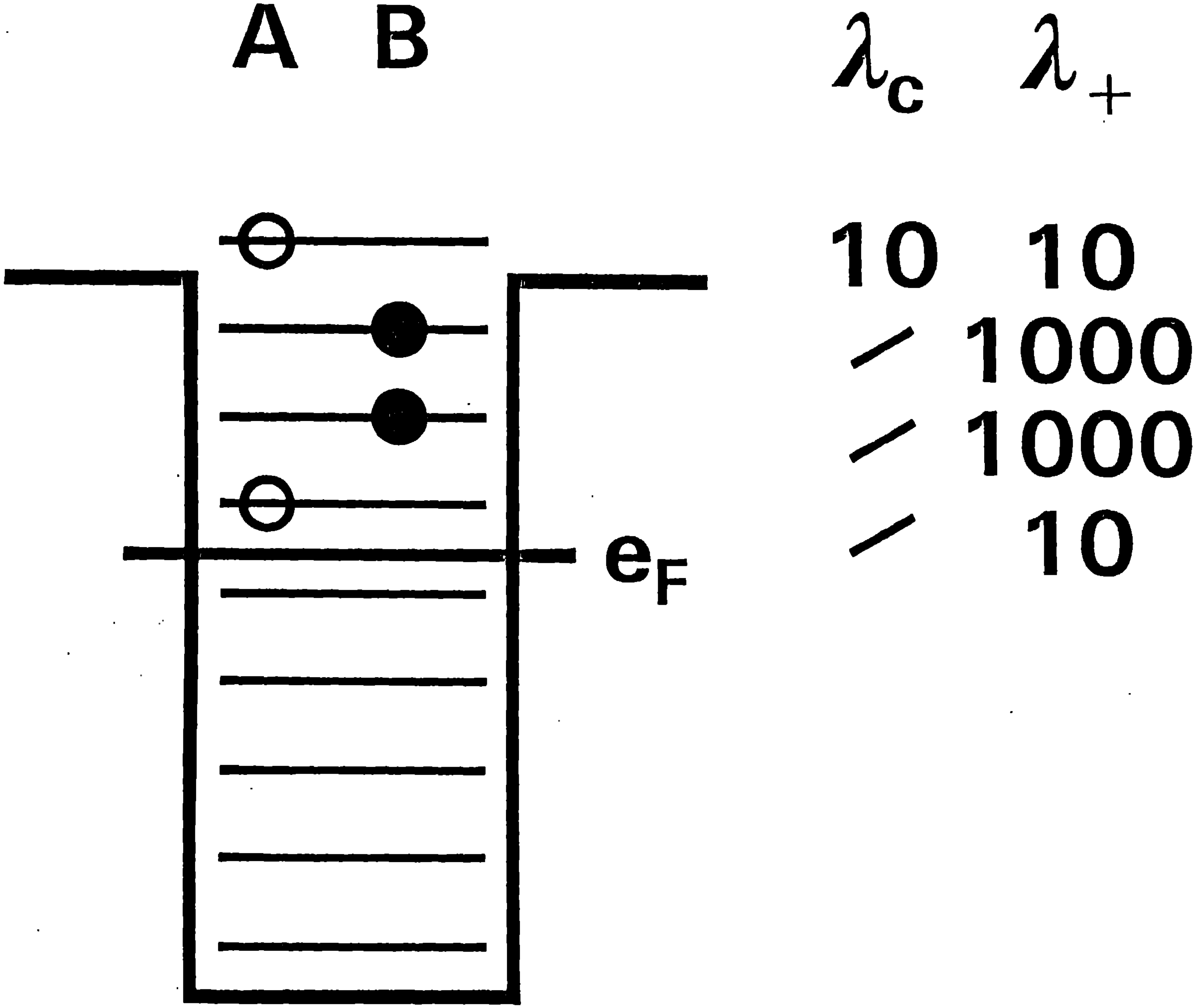
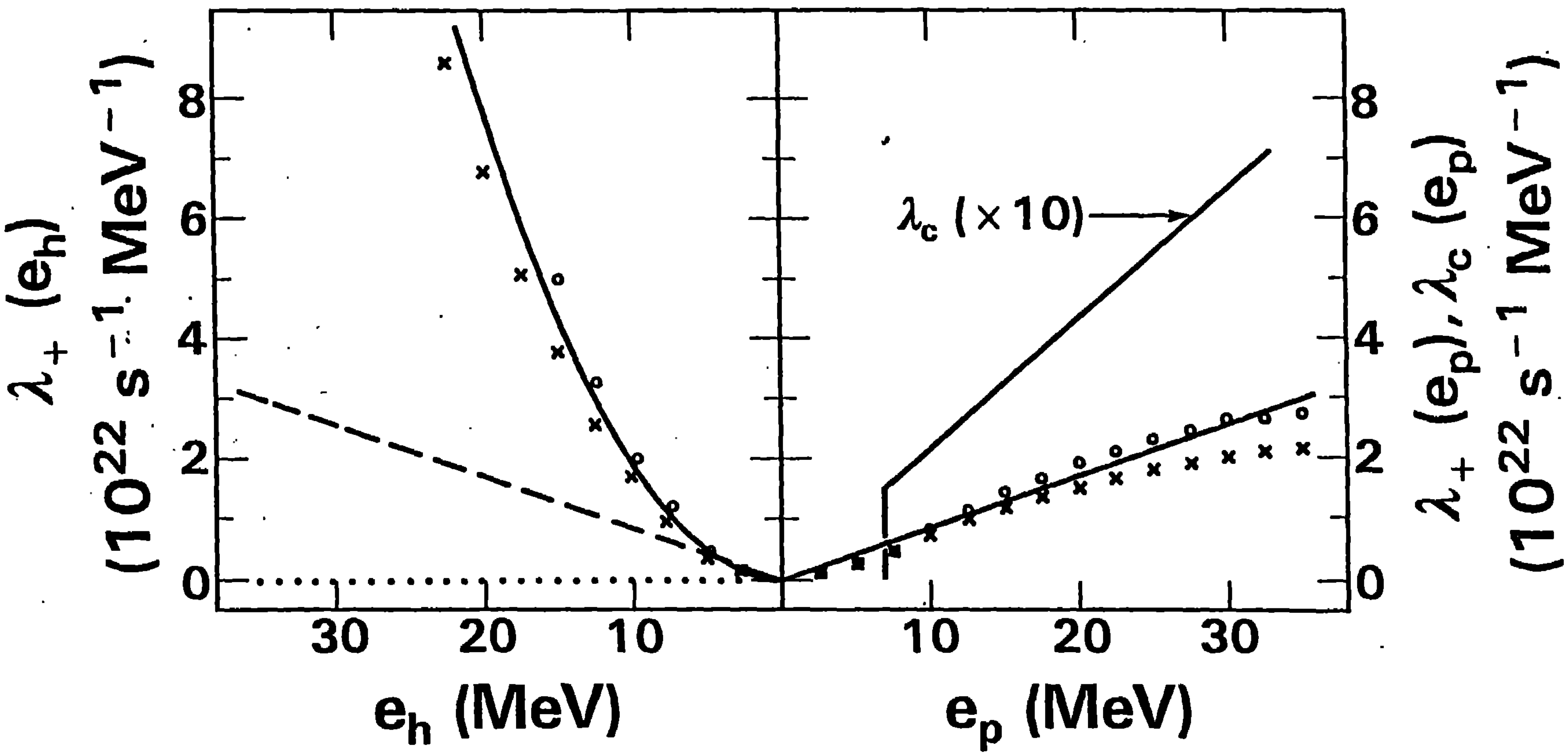


Figure 1



8
6
4
2
0
30
20
10
10
20
30
$\lambda_+ \ (e_h)$
$(10^{22}\ s^{-1}\ MeV^{-1})$
e_h (MeV)
e_p (MeV)
$\lambda_c\ (\times 10)$
$\lambda_+ \ (e_p), \lambda_c \ (e_p)$
$(10^{22}\ s^{-1}\ MeV^{-1})$

Figure 2

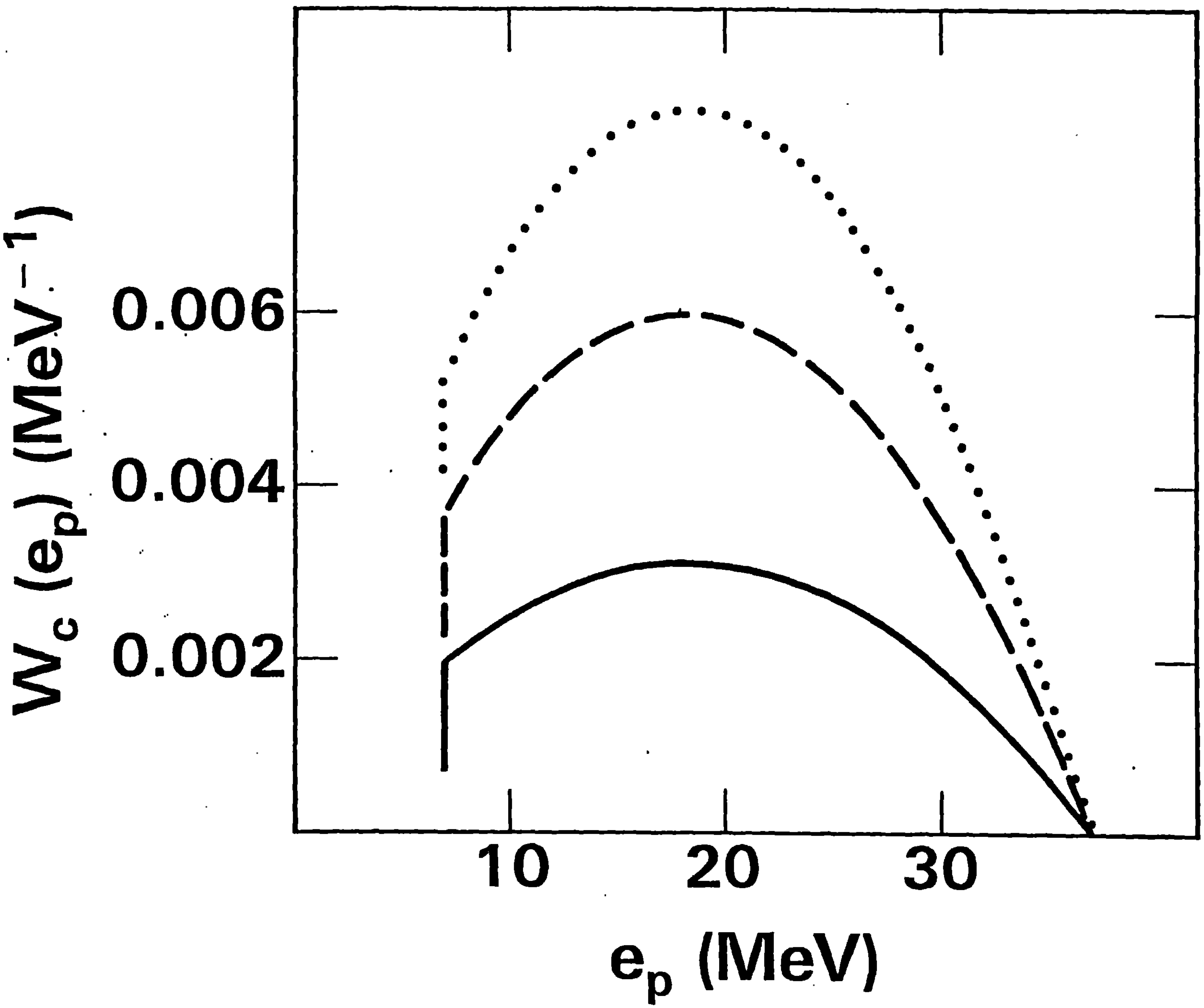


Figure 3

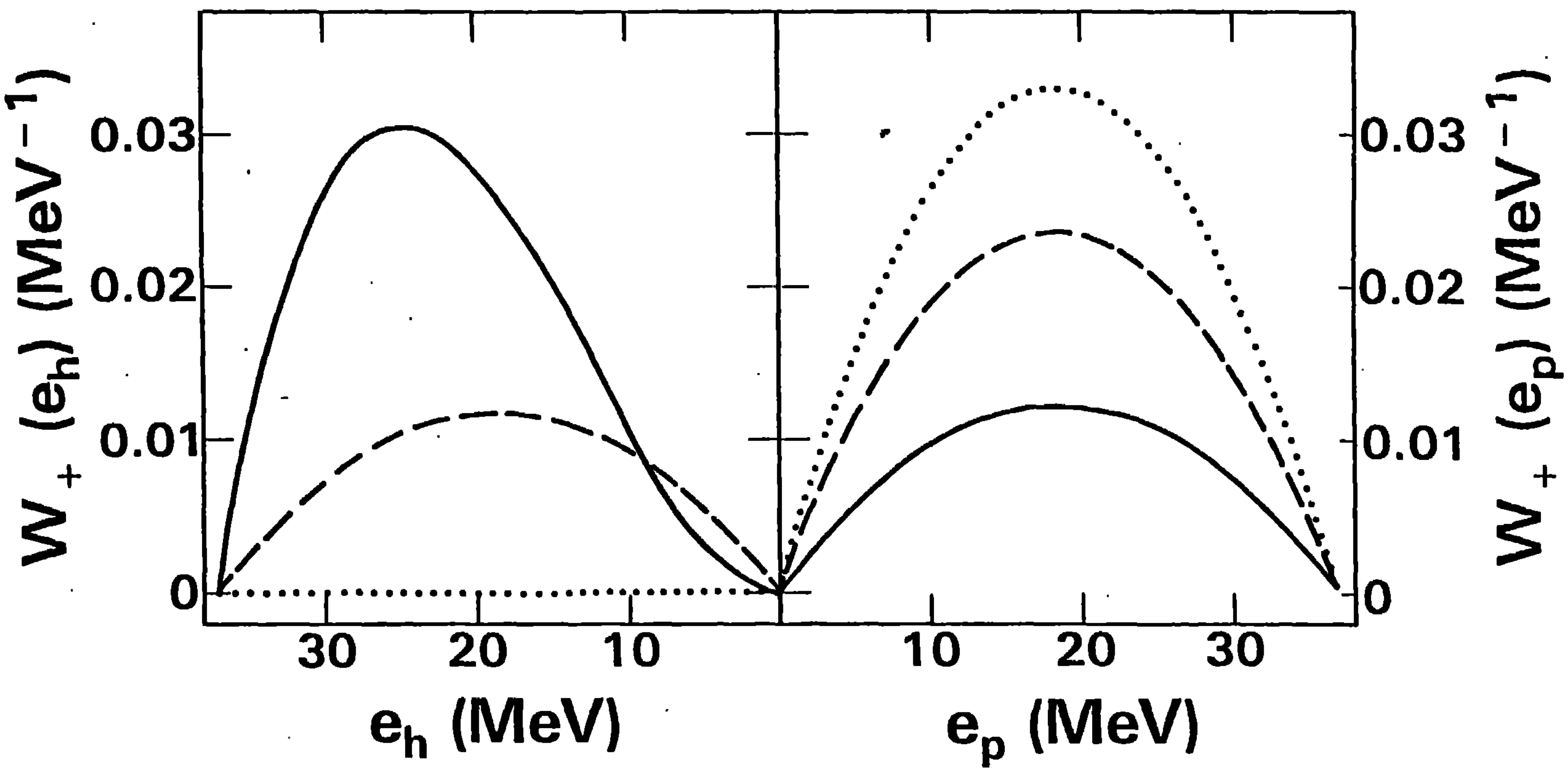
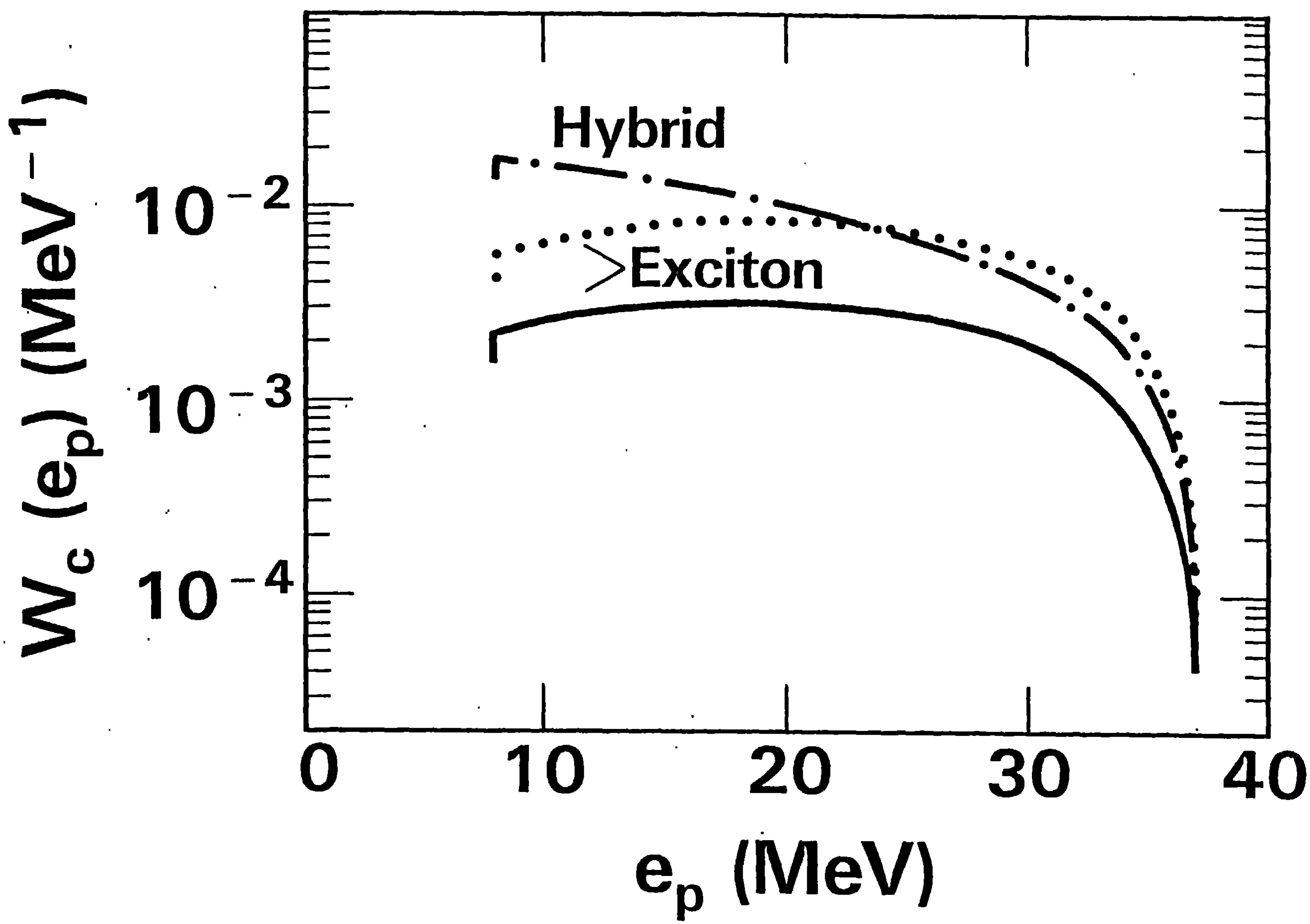


Figure 4



Hybrid
Exciton
10^{-2}
10^{-3}
10^{-4}
$W_c\ (e_p)\ (\mathrm{MeV}^{-1})$
0
10
20
30
40
e_p (MeV)

Figure 5

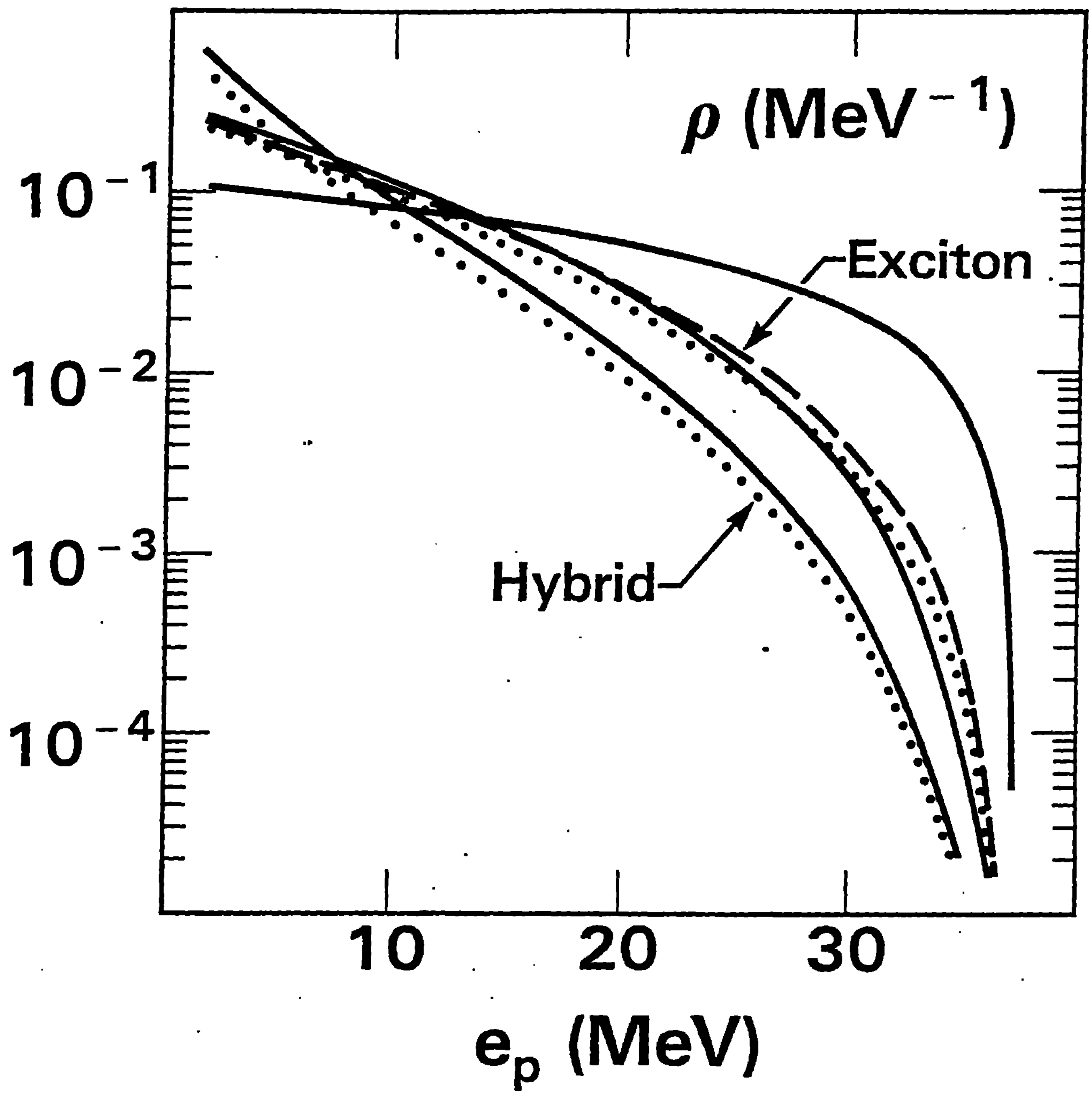


Figure 6

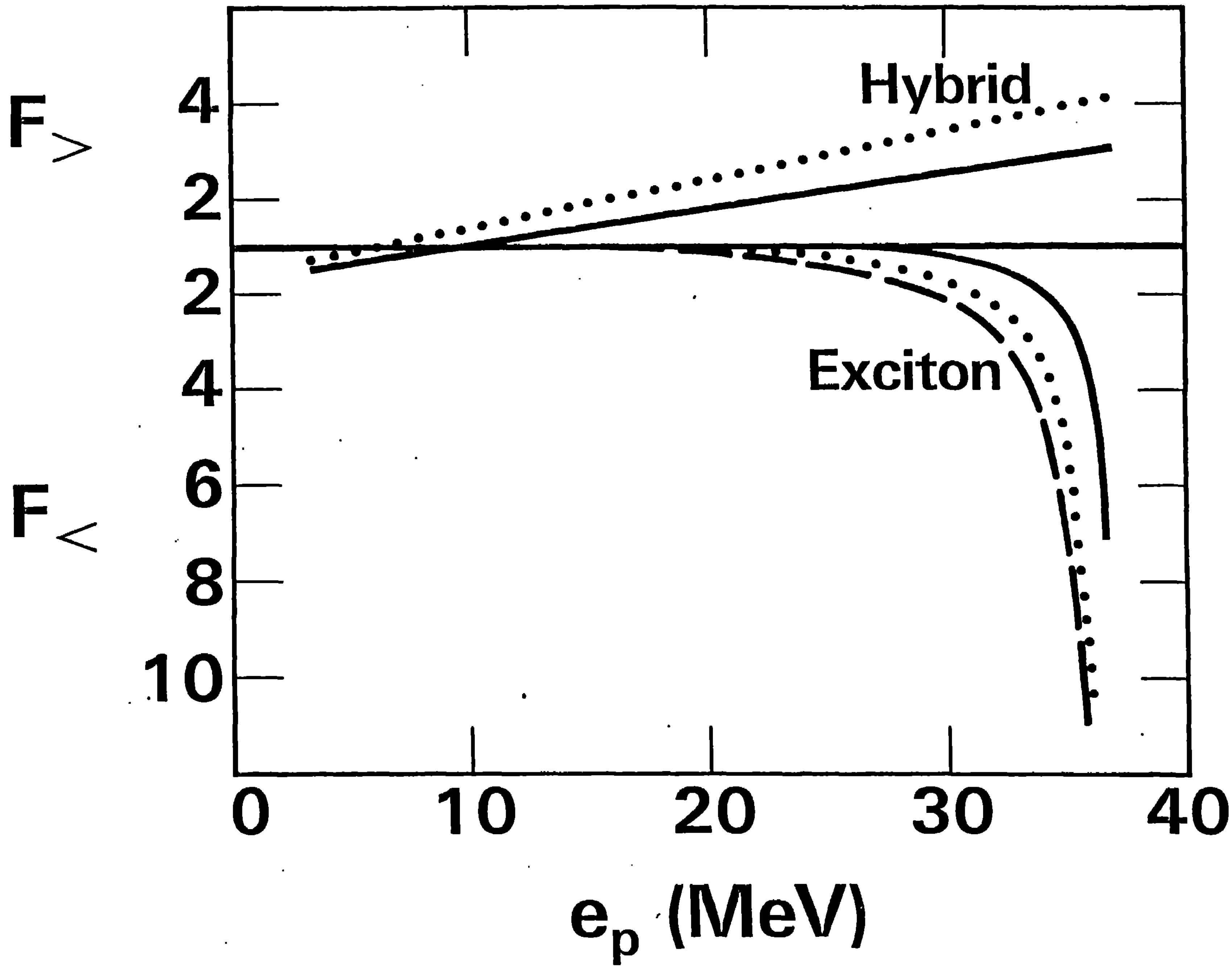


Figure 7

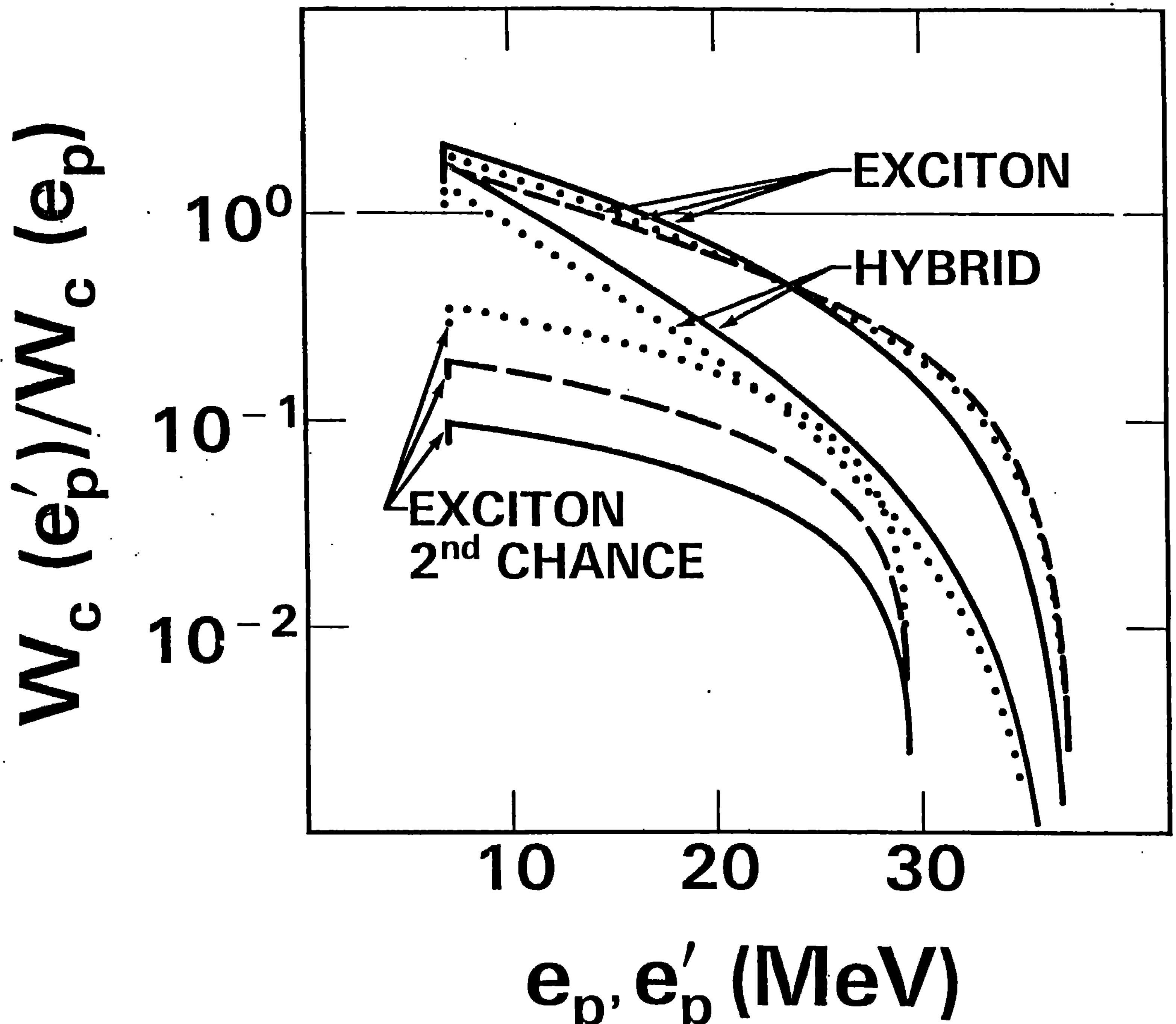

EXCITON
HYBRID
EXCITON
2nd CHANCE
10^0
10^{-1}
10^{-2}
10
20
30
$W_c(e'_p)/W_c(e_p)$
e_p, e'_p (MeV)

Figure 8

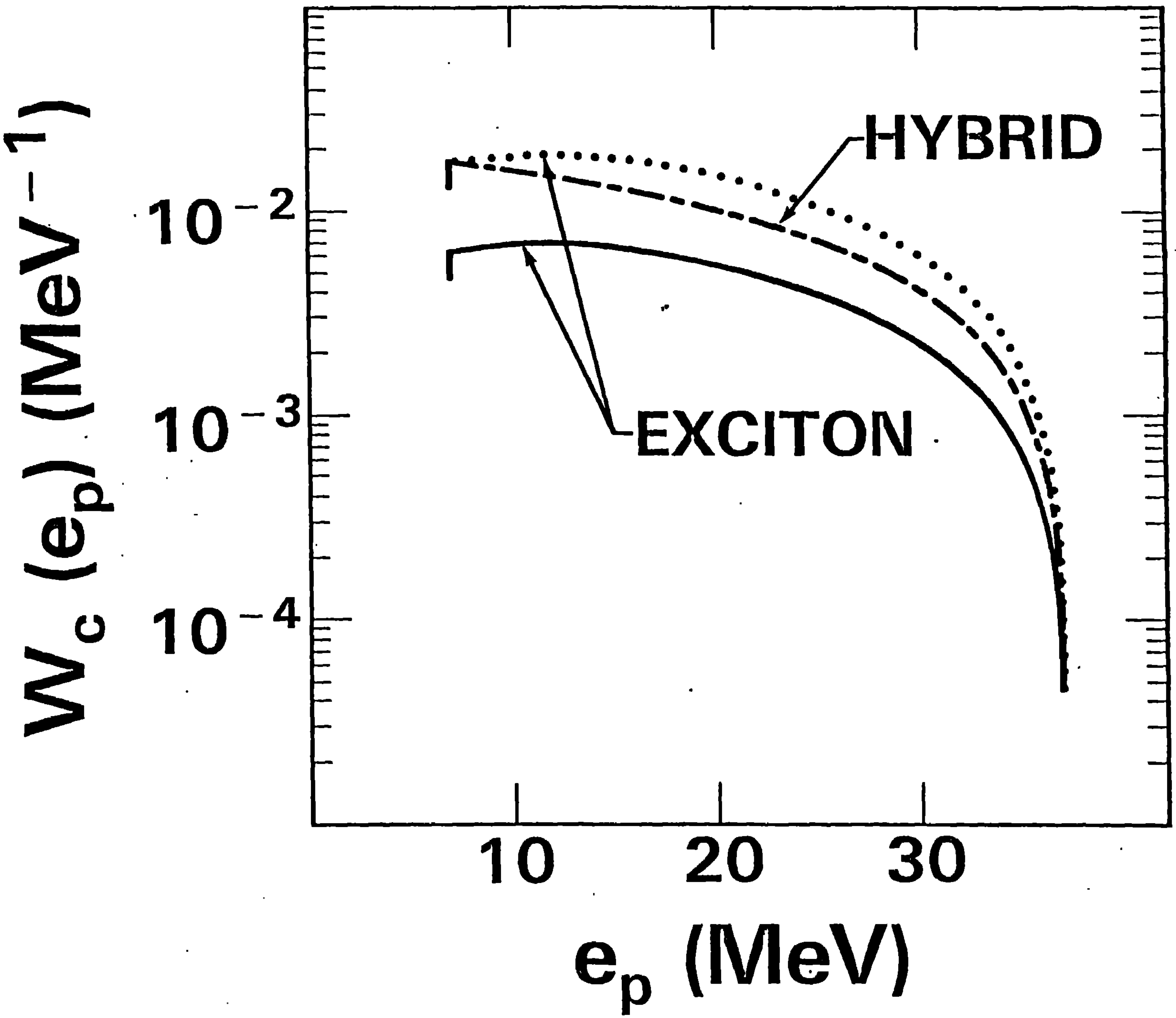


Figure 9